Bachelor Thesis Project - 2
Attitude Control of One DOF Drone

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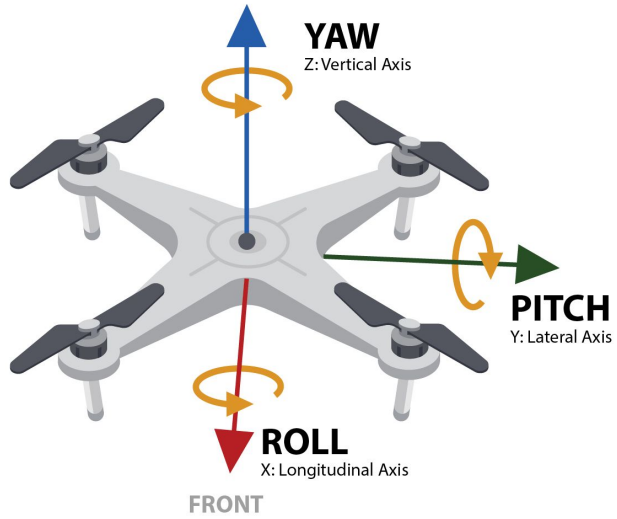
Introduction and Objective

Attitude control refers to the control of the orientation and position of an object in 3D space



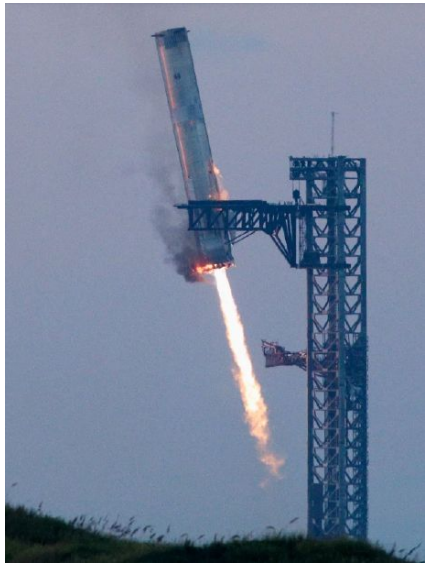
Attitude nomenclature

Applications:

- Coupled dynamics of aerial vehicles/submarines
- Balancing boards for rehabilitation
- Robots for maintaining orientation
- In defense for guided missiles

Learning Outcomes:

- Non-linear coupled dynamics control
- Nested loop controller
- Dynamics simulation



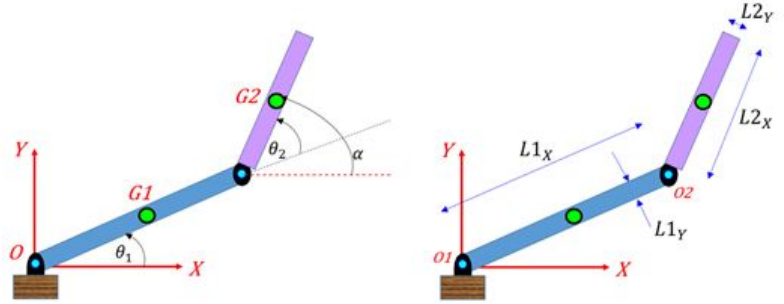
SpaceX rocket booster

MQ-9 Reaper of US Air Force

Understanding Simulation - 2R Robot Arm

Objective:

- Understanding dynamics derivation using Lagrange Equation
- Simulation of a real system using ode45
- Designing, implementing and tuning controllers for a given system

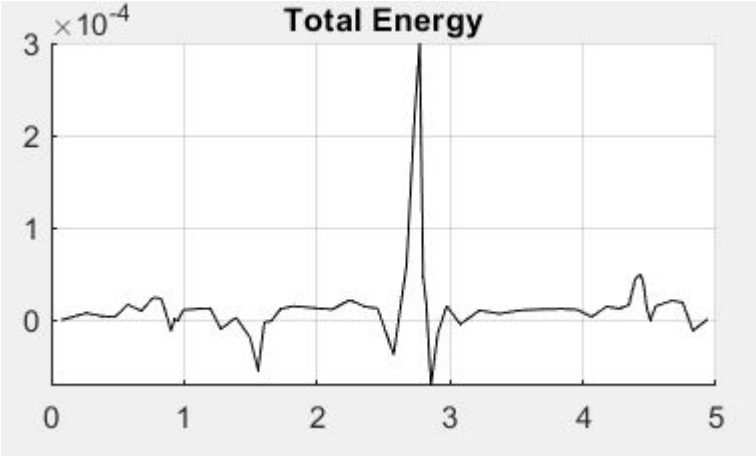


2R robot arm

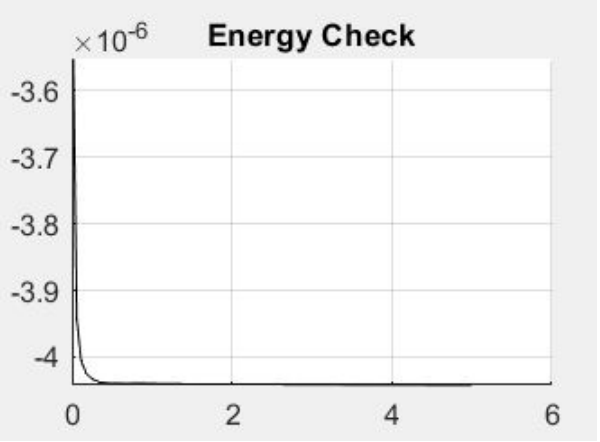
Kinematics	Energies	Dynamics
$V_{C_1} = \dot{\theta}_1 a_1(-\sin\theta_1 \mathbf{e_x} + \cos\theta_1 \mathbf{e_y})$ $V_{C_2} = (l_1/a_1)V_{C_1} + (\dot{\theta}_1 + \dot{\theta}_2)a_2(\cdot$ $-\sin(\theta_1+\theta_2)\mathbf{e_x} + \cos(\theta_1+\theta_2)\mathbf{e_y})$ $\omega_1 = \dot{\theta}_1 \qquad \omega_2 = \dot{\theta}_1 + \dot{\theta}_2$	$KE_1 = (1/2)(m_1 a_1^2 {\omega_1}^2 + I_1 {\omega_1}^2)$ $KE_2 = (1/2)(m_2(l_1^2\omega_1^2 + a_2^2\omega_2^2 + 2l_1a_2\omega_1\omega_2\cos\theta_2) + I_2\omega_2^2)$ $PE_1 = m_1 g a_1 \sin\theta_1$ $PE_2 = m_1 g(l_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2))$	$\mathbf{M}(\theta_1,\theta_2)\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix} + \mathbf{C}(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2)\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{pmatrix} + \mathbf{G}(\theta_1,\theta_2) = \begin{pmatrix}\tau_1\\ \tau_2\end{pmatrix}$
State Space		Controller
$\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \dot{\theta}_1\\ \dot{\theta}_2\end{pmatrix} = \begin{pmatrix}-M^{-1}C & 0\\ I_{2X2} & 0\end{pmatrix}\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \theta_1\\ \theta_2\end{pmatrix} + M^{-1}\begin{pmatrix}\tau_1\\ \tau_2\\ 0\\ 0\end{pmatrix} - M^{-1}\begin{pmatrix}G\\ 0\end{pmatrix}$		Proportional Integral Derivative Controller

Understanding Simulation - 2R Robot Arm

- ode45 was used to simulate the system and a PID controller was used for setpoint and trajectory tracking
- Momentum was calculated both, from torque integration and velocities for verification



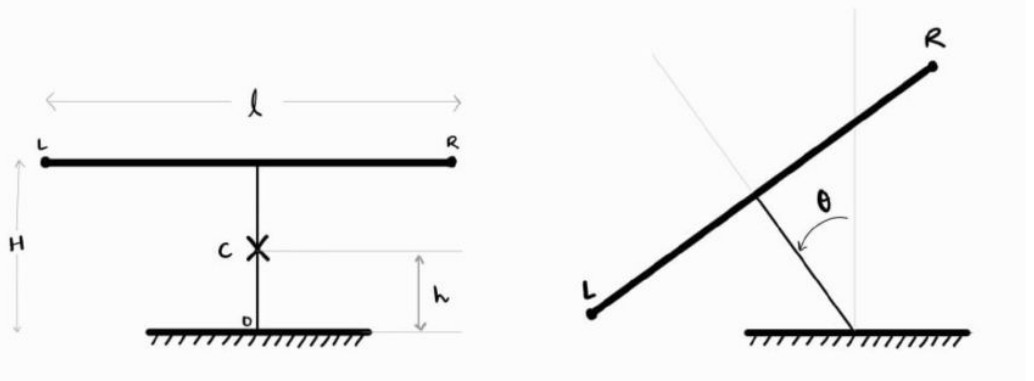
Total energy of a freely falling system



Difference in total energy calculated from momentum integration and kinematics

1D balancing - Mechanics

A one dimensional balancing robot free to rotate about ground contact point



Kinematics:

$$\vec{r}_C = -h(\sin\theta\hat{i} - \cos\theta\hat{j})$$
$$\dot{\vec{r}}_C = \vec{v}_C = -h\dot{\theta}(\cos\theta\hat{i} + h\sin\theta\hat{j})$$
$$\omega = \dot{\theta}$$

Dynamics:

Kinetic Energy: $KE = \frac{I_0\omega^2}{2} = \frac{I_0\dot{\theta}^2}{2}$

Potential Energy: $PE = Mgh\cos\theta$

Lagrange: $L = KE - PE$

Euler Lagrange Equation: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \tau$

Upon solving,

$$I_0\ddot{\theta} - Mgh\sin\theta = \frac{(u_R - u_L)l}{2}$$

1D balancing - Mechanics

Constraints:

Robot is always in touch with the ground

$$(u_L + u_R)\cos\theta < \kappa M g \qquad \kappa \leq 1$$

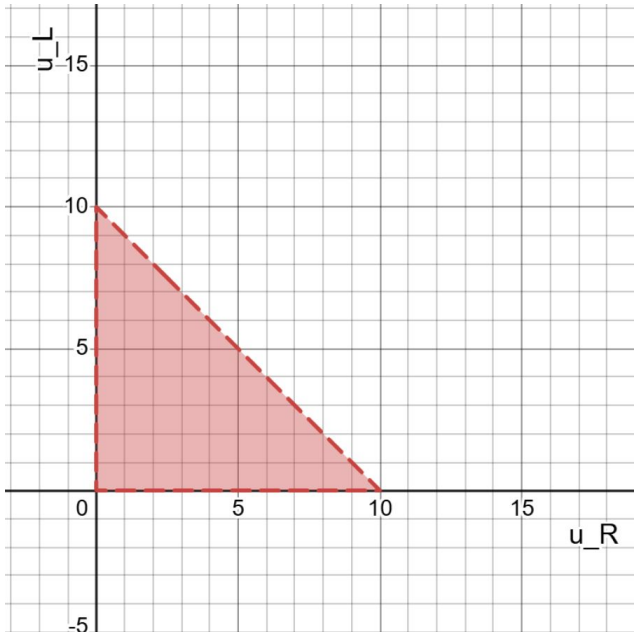
Only one point of the robot is in contact with the ground

$$|\theta| < \frac{\pi}{2} - \tan^{-1}(l/2H)$$

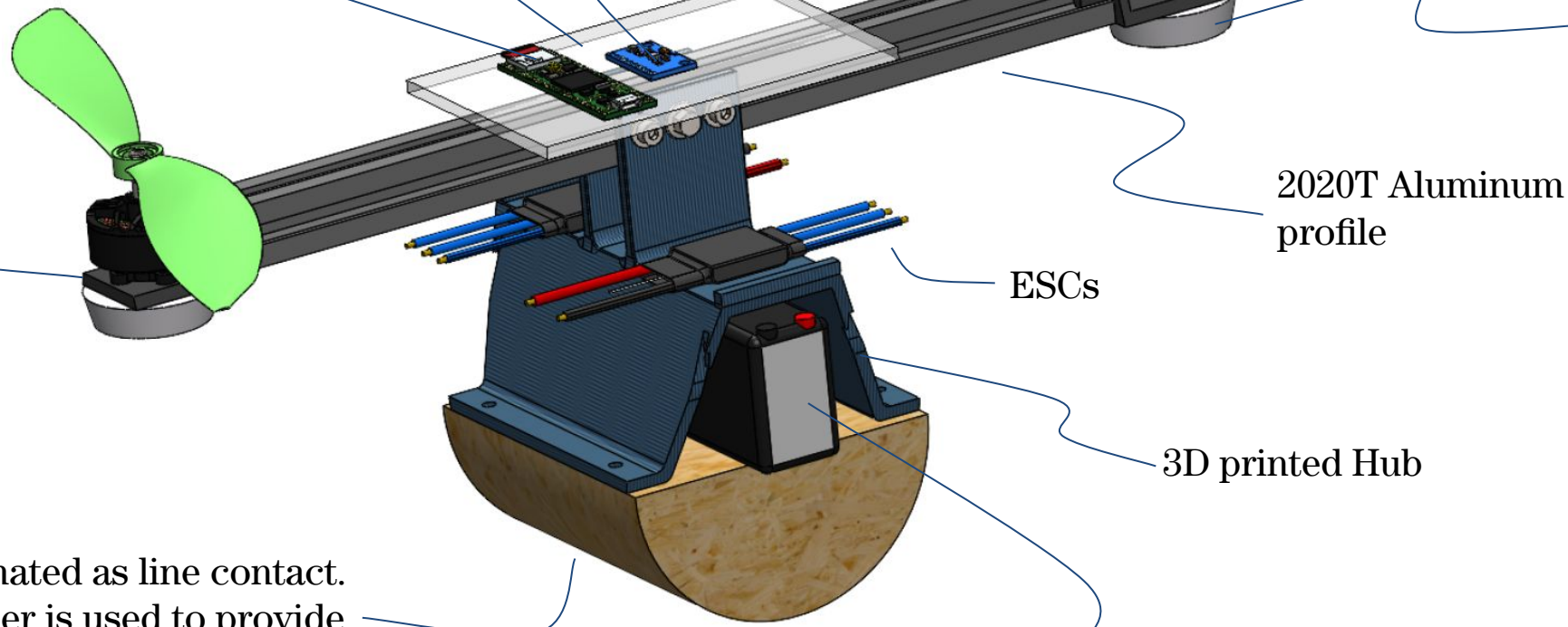
Propellers can produce thrust in one direction

$$u_L, u_R > 0$$

Solution Space:

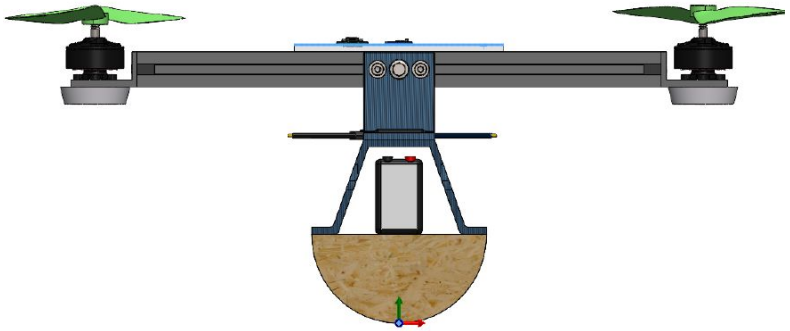


Physical Model

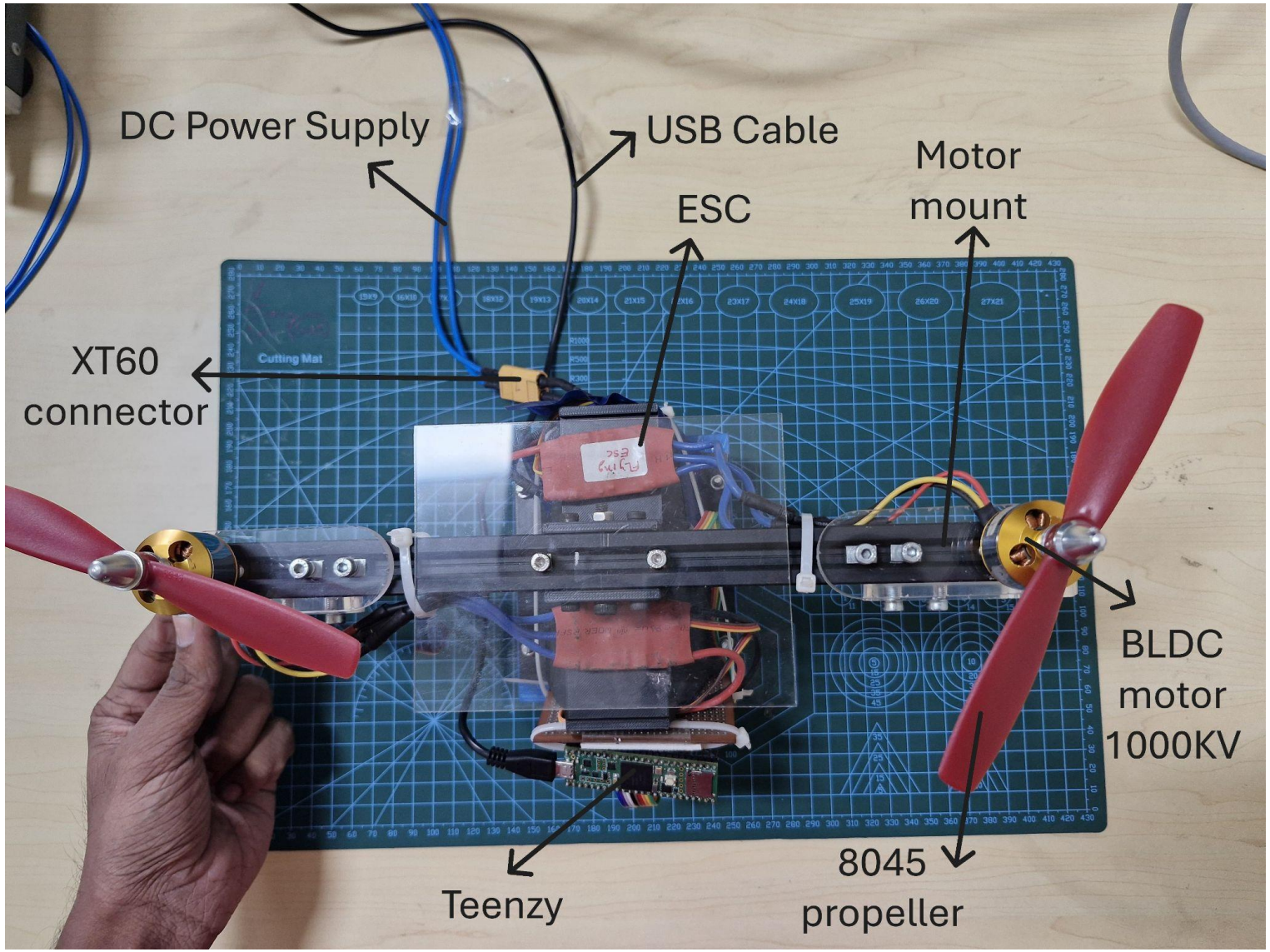


Physical parameters

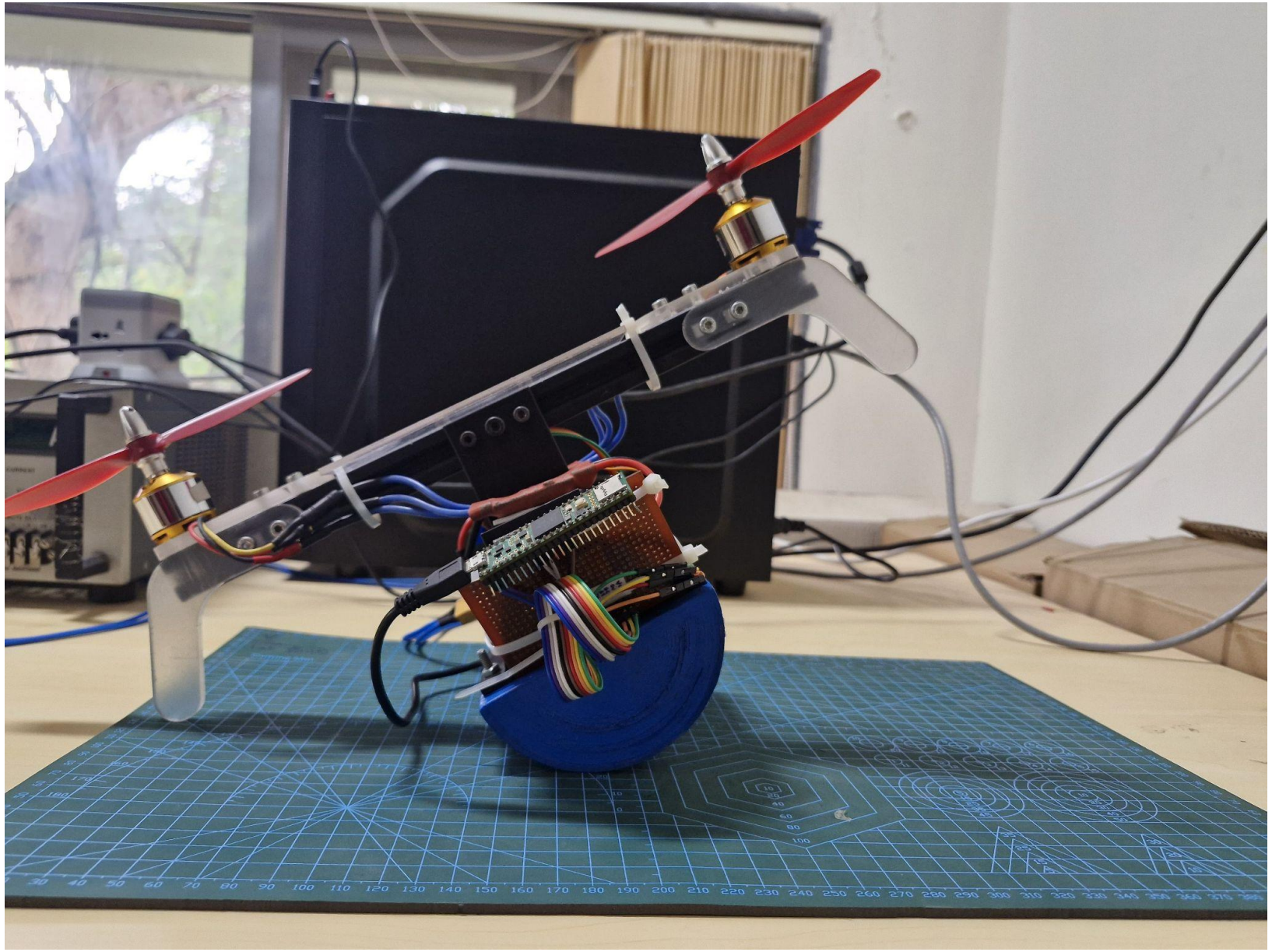
Description	Notation	Units	Current Value
Distance between propellors	l	mm	348
Height of COM above the ground	h	mm	90
Height of the lowest point of the motor-motor mount assembly	H	mm	124
Moment of inertia about COM through nominal axis	I	Kg/m^2	0.005
Mass	M	Kg	1.05



Hardware


DC Power Supply
USB Cable
ESC
Motor mount
XT60 connector
BLDC motor 1000KV
Teenzy
8045 propeller

Hardware



Controller

PD controller:

Error dynamics $$\ddot{e} + K_d\dot{e} + K_p e = 0$$

Commanded acceleration $$\ddot{\theta}_{des} = \ddot{\theta}_t - (K_p e + K_d \dot{e})$$

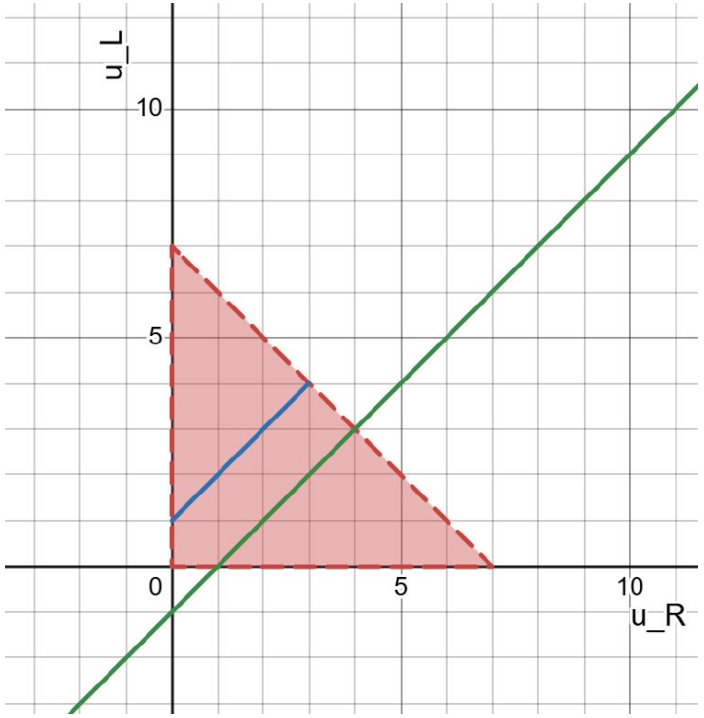
Commanded thrust $$u_R - u_L = u_0 \qquad u_0 := u_d - \alpha_1 u_m \sin\theta$$

$$u_d := 2I_0\ddot{\theta}_{des}/l$$
$$u_m := \kappa M g$$
$$\alpha_1 := 2h/\kappa l$$

With Constraints:

Taking into account:

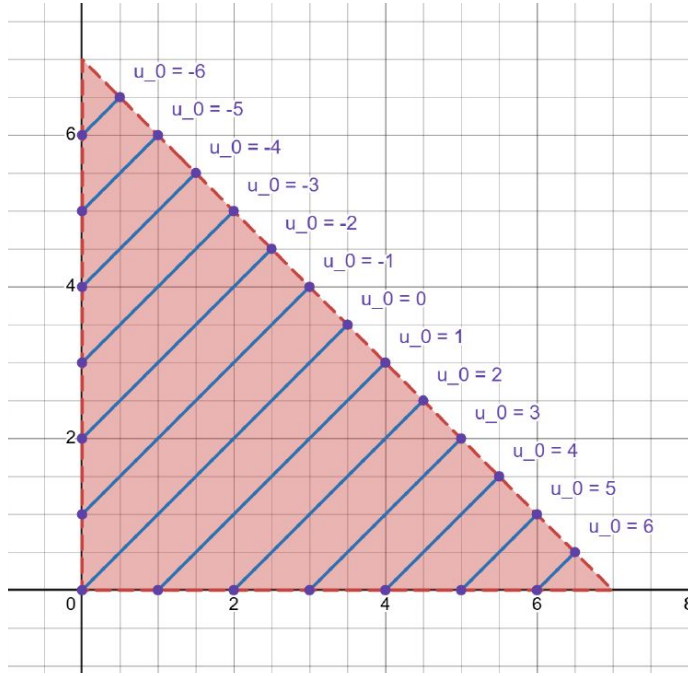
1. Normal Contact > 0
2. Thrust > 0



Green line is a solution of the PD controller without constraints. Blue line is a solution of the PD controller with constraints

$$0 < u_R < \frac{u_m \sec\theta + u_0}{2}$$

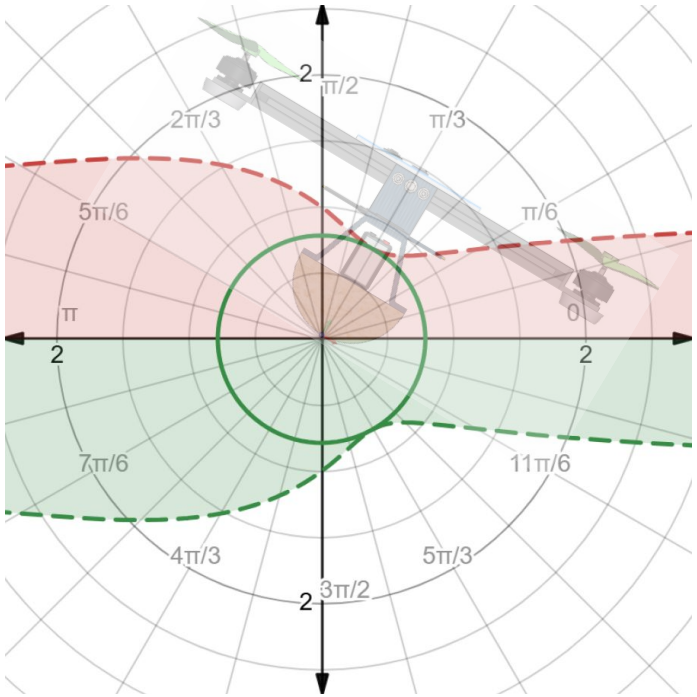
$$0 < u_L < \frac{u_m \sec\theta - u_0}{2}$$



Solution space for varying u_0

Controller: Existence of a solution

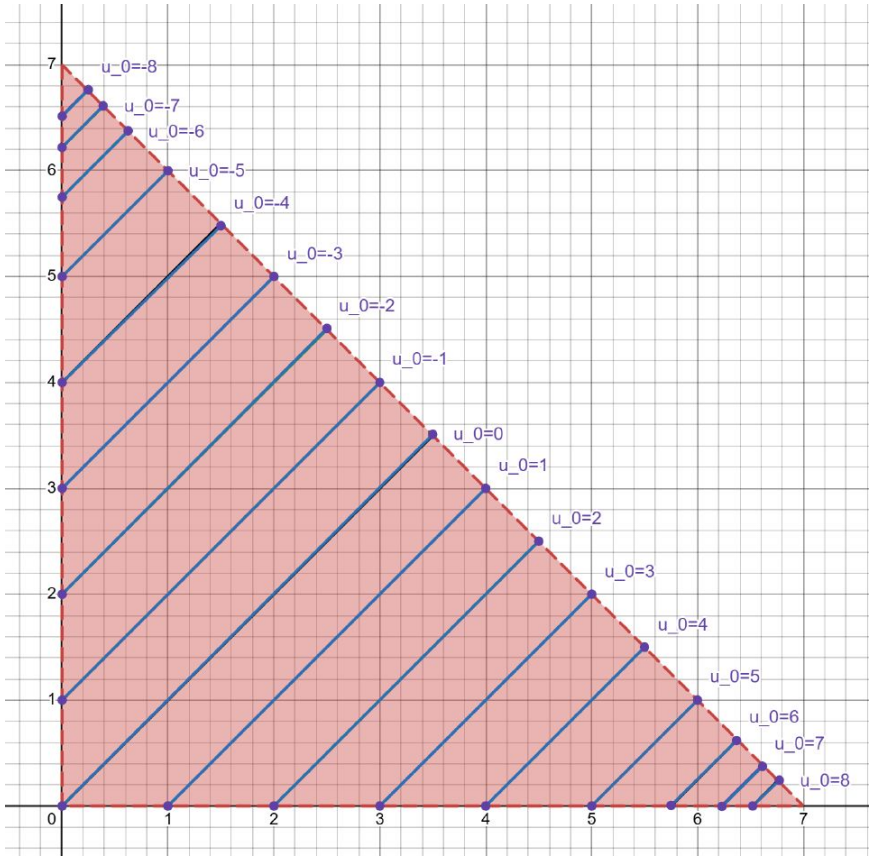
Since u_d depends only on controller gains and trajectory inputs, we try to constrain them so that the controller is always able to find a solution



Solution space of u_d as a function of time

- For a given orientation, the red and green lines represent the maximum and minimum possible values of u_d permissible
- To guarantee a solution, we will restrict u_d to inside the green circle
- Analytical solution for radius of the green circle does not exist
- With numerical estimates and trajectory acceleration bounds, estimate Kp and Kd

Saturation of thrust is required



Required saturation of u_0

Controller: Saturation

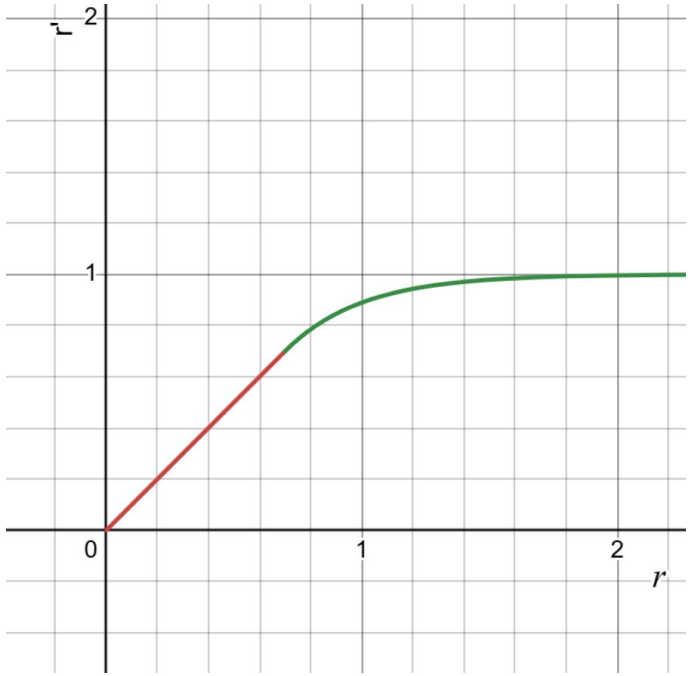
u_0 is saturated the prevent individual thrusts to cross the constraints. $r = |u_0 \cos\theta / u_m|$

Saturation function requirements:

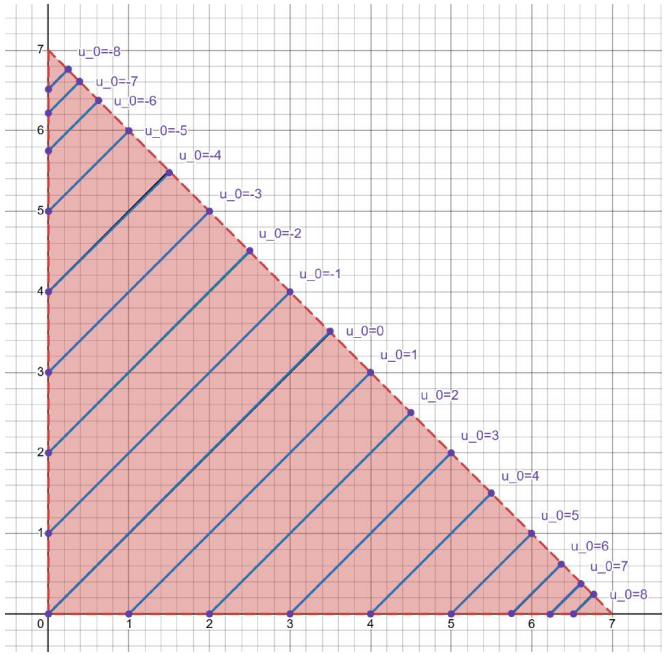
- Linear to a large extent
- C0 and C1 continuity for smooth trajectories
- Asymptotically approaching saturation limits

Exponential function is chosen with linearity in central region

$$r' = \begin{cases} r & 0 \le r \le \alpha_2 \\ 1 - (1-\alpha_2)e^{\frac{(\alpha_2 - r)}{1-\alpha_2}} & r > \alpha_2 \end{cases}$$



Exponential saturation function

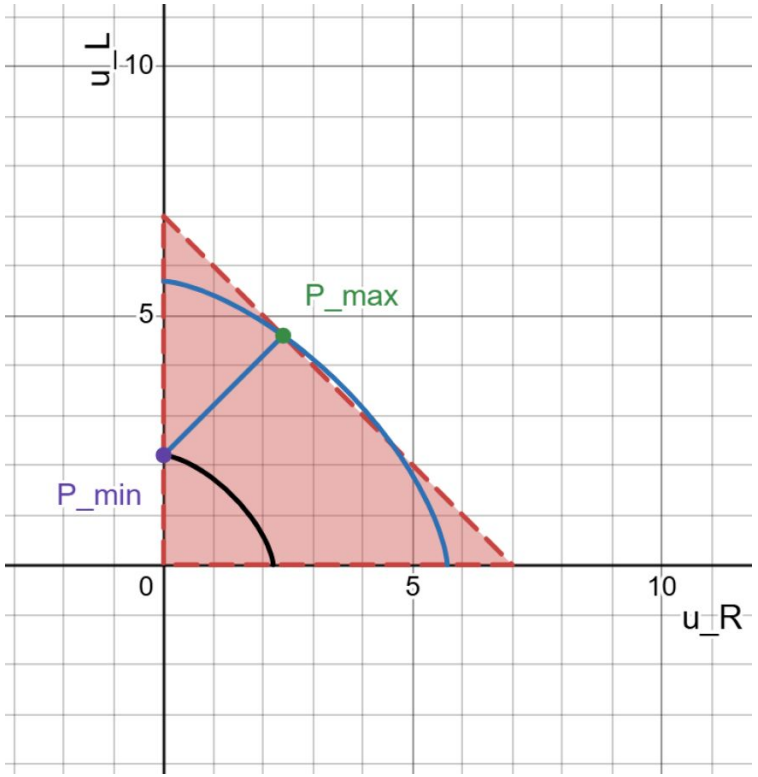


Exponential saturation of u_0

Controller: Selecting Optimal Gains

Optimizing power requirements:

$$P = \frac{k_T}{k_F^{3/2}}(u_R^{3/2} + u_L^{3/2})$$

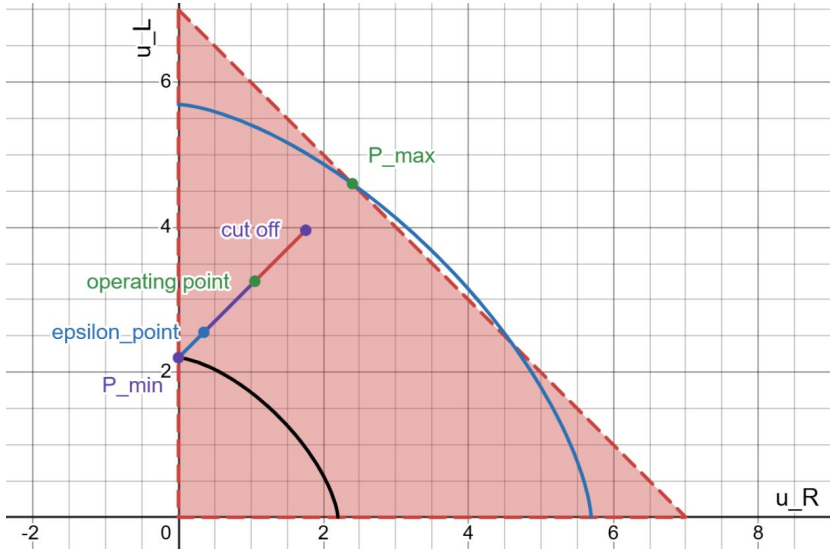


Minimum and maximum power points

- We want atleast ε fraction of power in each motor for propellers to run
- We want atleast β fraction of power to be available on each side of the operating point

$$\epsilon + 2\beta < 1$$

- Since power is non-linear in thrusts and interpolating power to get thrust is computationally heavy, we will approximate our power requirements to vary linearly with thrusts



Identification of operating point

$\epsilon = 0.15, \beta = 0.3$

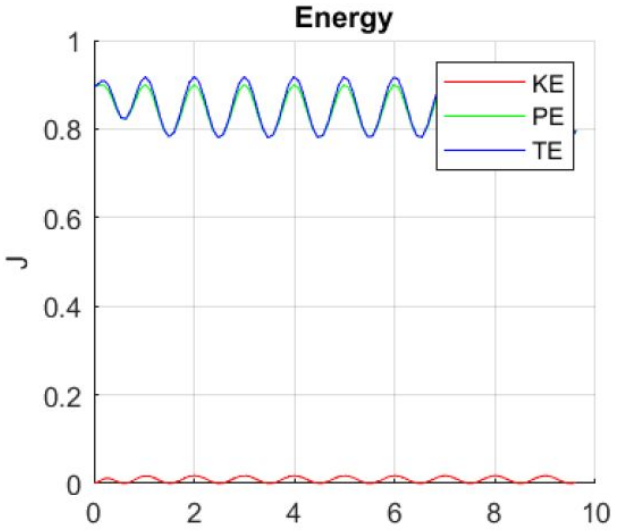
Controller implementation

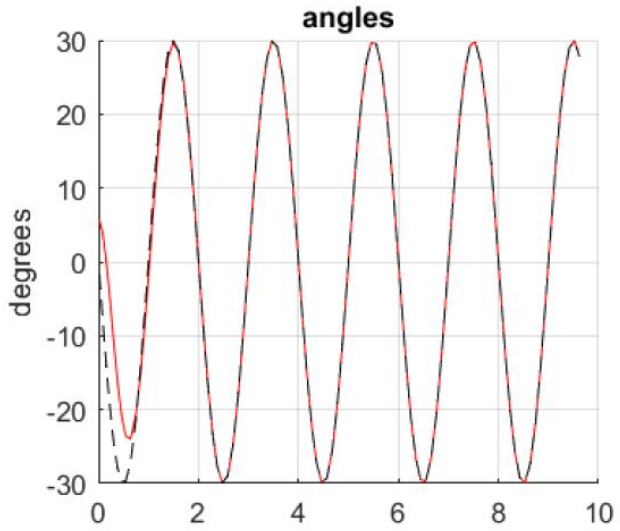
$$PID = -(K_p * e + K_i \int edt + K_d \dot{e})$$

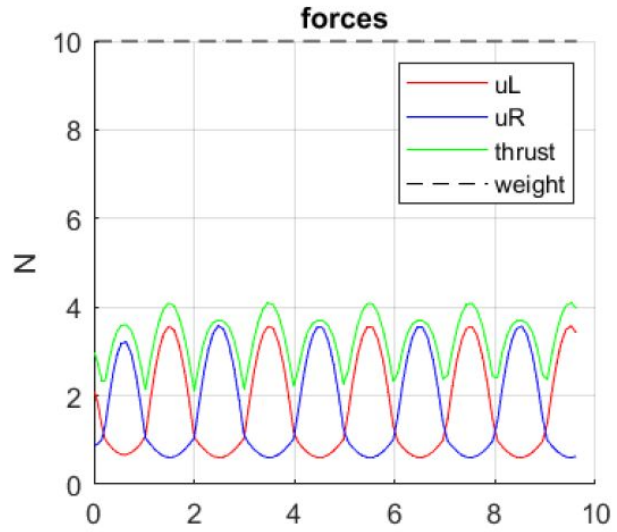
$$\omega_L = max(0, -PID)$$

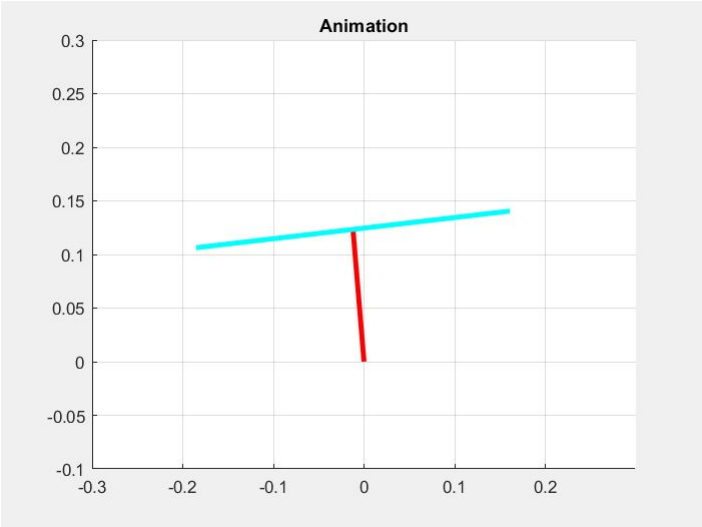
$$\omega_R = max(0, PID)$$

Results: Simulation







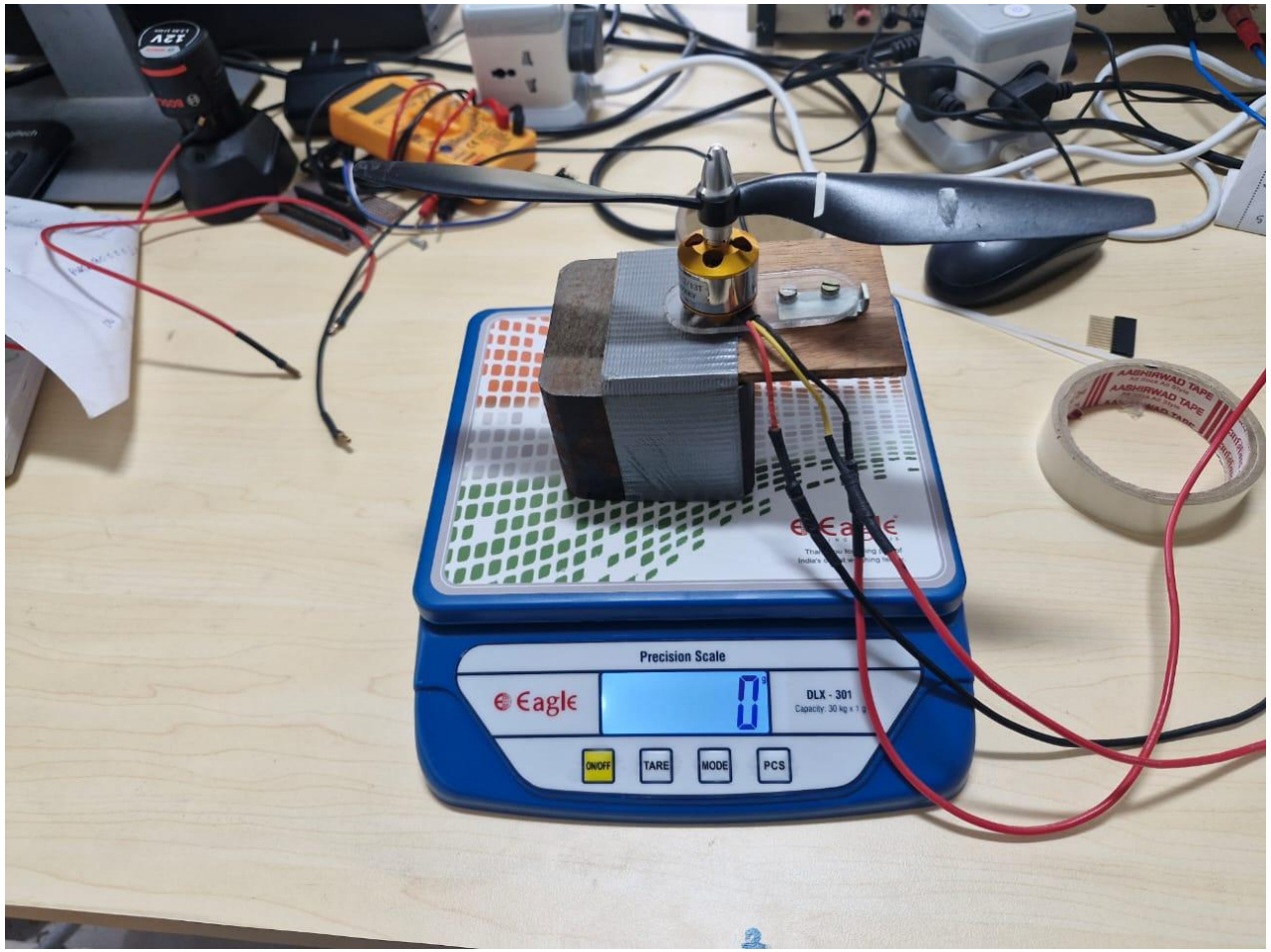


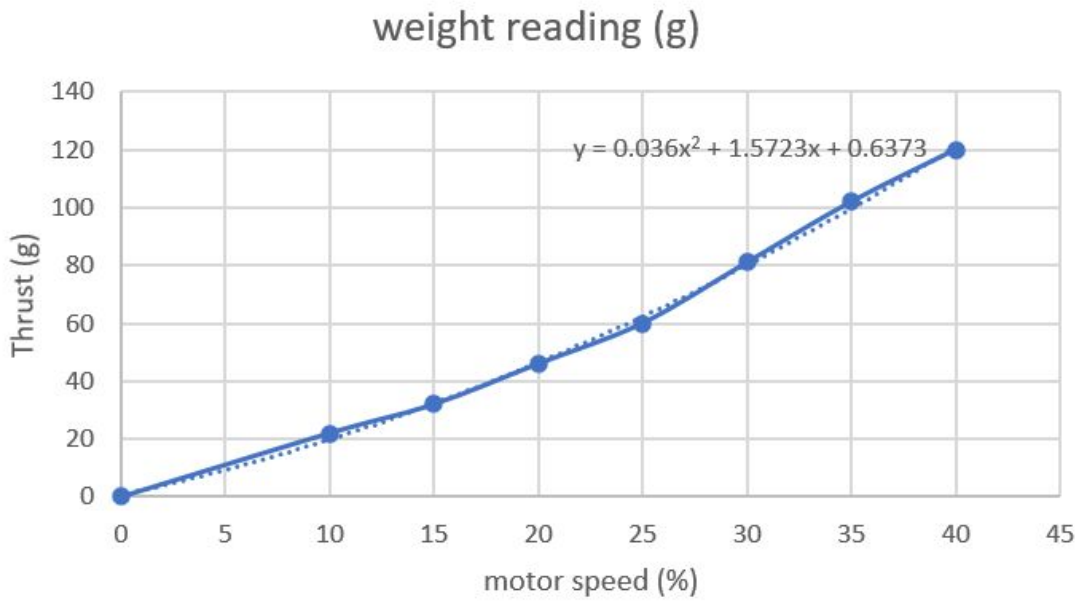
Results from trajectory tracking simulation in MATLAB:

- ode45 used for system simulation
- 0.5Hz sinusoidal trajectory requirement
- Non zero initial conditions

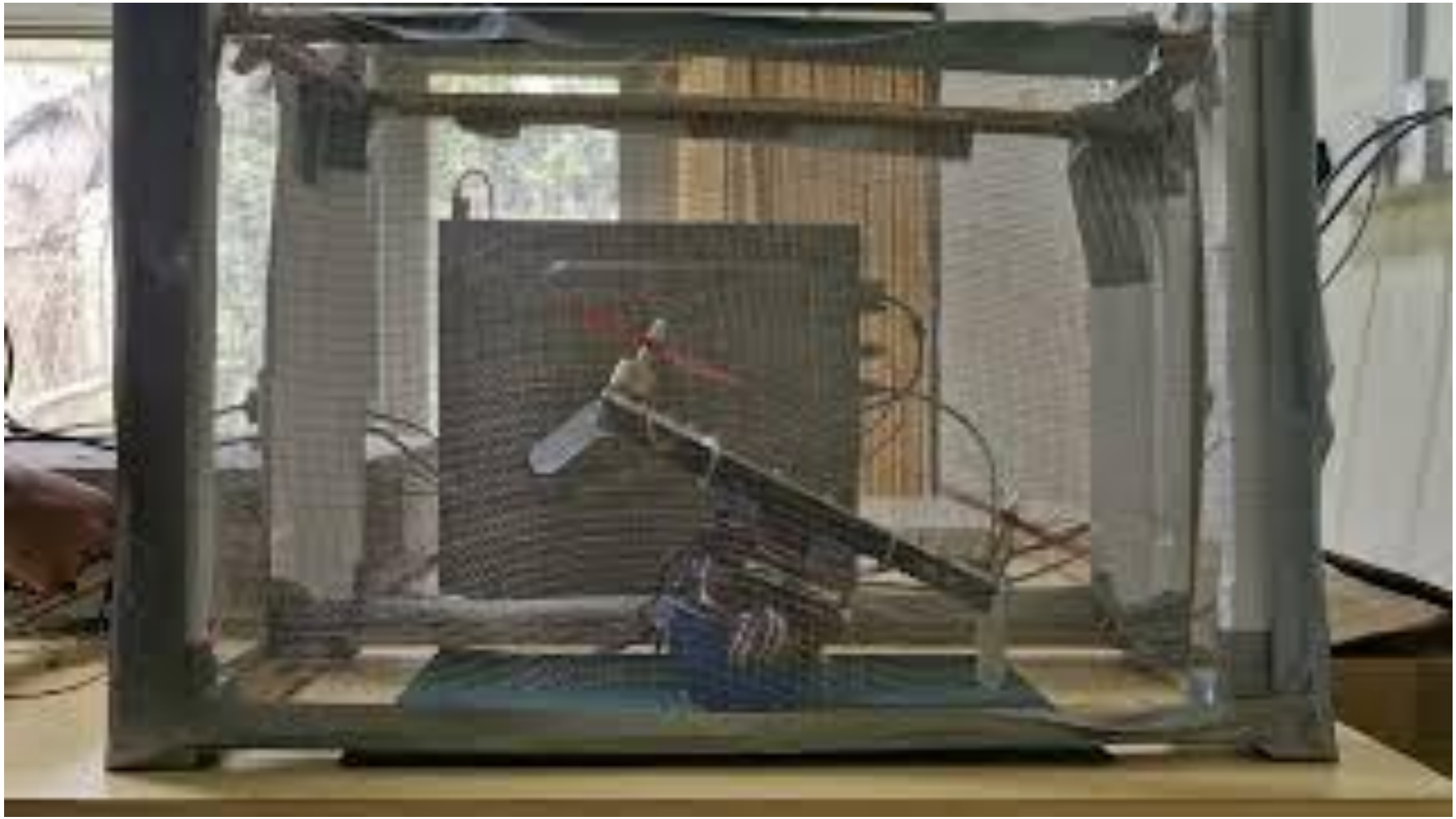
- We observe thrusts are always positive and much lesser than weight
- Thrusts get saturated for low requirements
- Continuous variation of thrusts but with abrupt change in jerk
- Kinetic energy has lower magnitude oscillations compared to potential energy
- Tracking catches up in ~1s

Results: Static Thrust Test

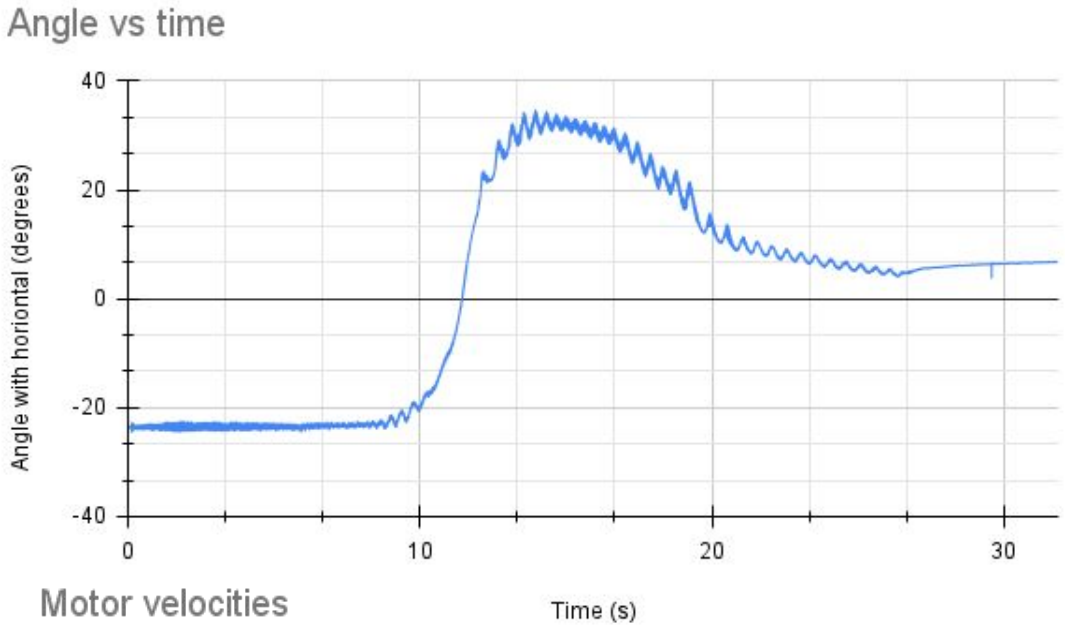


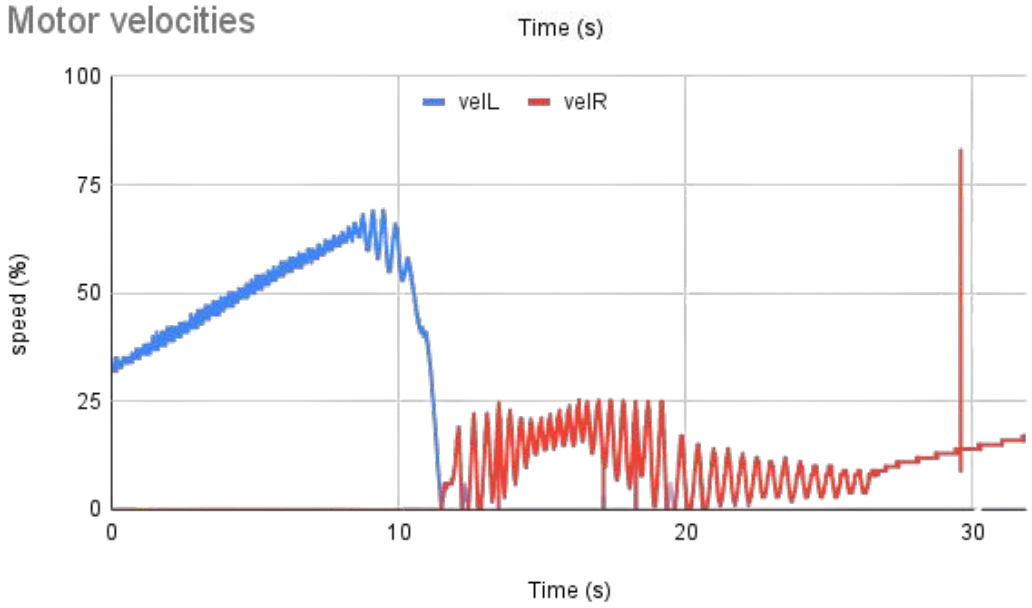


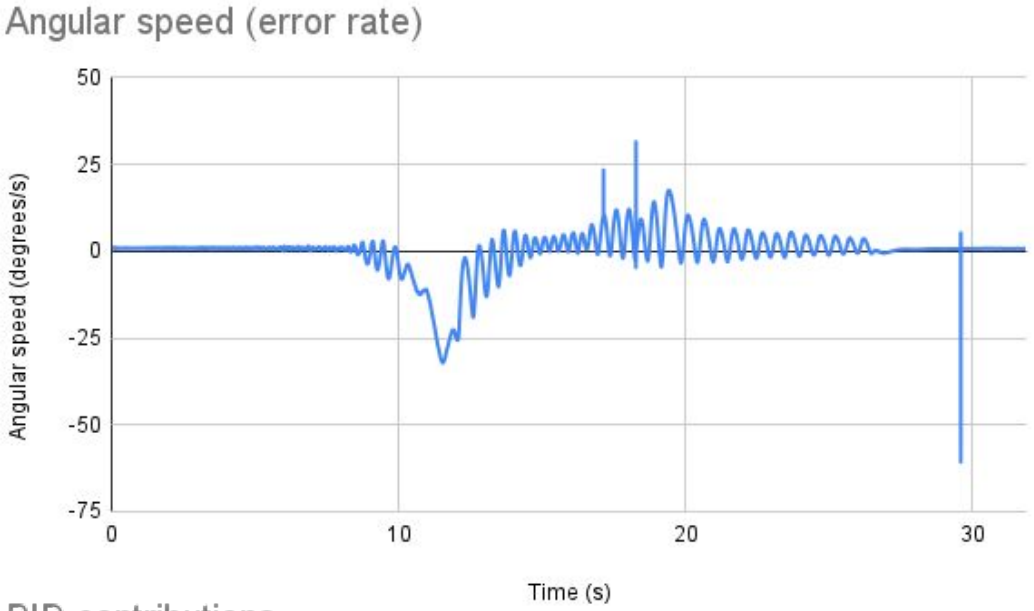
Results: Hardware

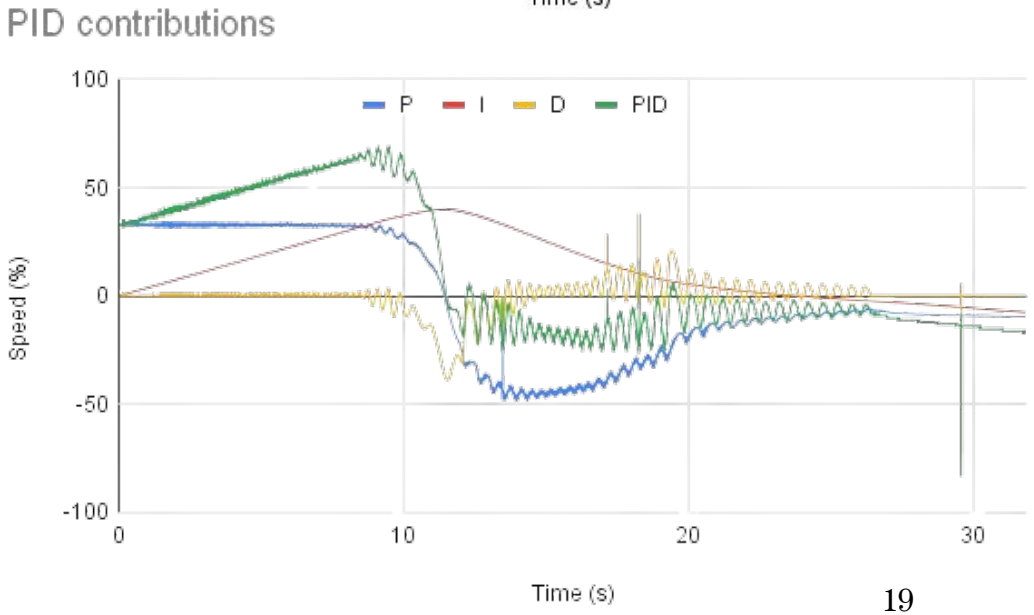


Results: Hardware









Future Work

- Increase Propeller size
- Use Simulated Control Law
- Implement Yaw Control

Understanding Simulation - 2R Robot Arm

Dynamics:

$$\mathbf{M}(\theta_1,\theta_2)\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix}+\mathbf{C}(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2)\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{pmatrix}+\mathbf{G}(\theta_1,\theta_2)=\begin{pmatrix}\tau_1\\ \tau_2\end{pmatrix}$$

$$\mathbf{M}(\theta_1,\theta_2)=(I_2+m_2a_2^2)I_{2X2}+\begin{pmatrix}I_1+m_1a_1^2+m_2(l_1^2+2l_1a_2\cos(\theta_2)) & m_2l_1a_2\cos(\theta_2)\\ m_2l_1a_2\cos(\theta_2) & 0\end{pmatrix}$$

$$\mathbf{C}(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2)=m_2l_1a_2\sin\theta_2\begin{pmatrix}-\dot{\theta}_2 & -(\dot{\theta}_1+\dot{\theta}_2)\\ \dot{\theta}_1 & 0\end{pmatrix}\quad \mathbf{G}(\theta_1,\theta_2)=m_2ga_2\cos(\theta_1+\theta_2)(\mathbf{e_x}+\mathbf{e_y}+g\cos\theta_1\begin{pmatrix}m_1a_1+m_2l_1\\ 0\end{pmatrix}$$

State Space:

$$\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \dot{\theta}_1\\ \dot{\theta}_2\end{pmatrix}=\begin{pmatrix}-M^{-1}C & 0\\ I_{2X2} & 0\end{pmatrix}\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \theta_1\\ \theta_2\end{pmatrix}+M^{-1}\begin{pmatrix}\tau_1\\ \tau_2\\ 0\\ 0\end{pmatrix}-M^{-1}\begin{pmatrix}G\\ 0\end{pmatrix}$$

ode45 was used to simulate the system and a PID controller was used for setpoint and trajectory tracking.

Momentum was calculated both, from torque integration and velocities for verification